

Quantizing magnetic field and quark-hadron phase transition in a neutron star

Debades Bandyopadhyay,¹ Somenath Chakrabarty,² and Subrata Pal¹

¹*Saha Institute of Nuclear Physics, 1/AF Bidhannagar, Calcutta 700 064, India*

²*Department of Physics, University of Kalyani, Kalyani 741235, India and IUCAA, P.B. 4, Ganeshkhind, Pune 411007, India*

We investigate the influence of a strong magnetic field on various properties of neutron stars with quark-hadron phase transition. The one-gluon exchange contribution in a magnetic field is calculated in a relativistic Dirac-Hartree-Fock approach. In a magnetic field of 5×10^{18} G in the center of the star, the overall equation of state is softer in comparison to the field-free case resulting in the reduction of maximum mass of the neutron star.

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The matter density in the core of a neutron star could exceed up to a few times the nuclear matter saturation density. At such high density, it is expected that the quark degrees of freedom would be manifested. In fact, quark matter composed of comparable proportions of up, down and strange quarks has been conjectured [1,2] to be the true ground state of QCD at finite baryon density. Therefore, at such high baryon density, a transition from nuclear matter to a stable quark matter is a possibility. Several authors [3–6] have studied the effect of this phase transition on neutron star properties.

The presence of strong magnetic fields in neutron stars might have interesting astrophysical implications. Large magnetic field $B_m \sim 10^{14}$ G has been estimated at the surface of neutron stars [7]. On the other hand, in the core, the field may have been amplified considerably due to flux conservation from the original weak field of the progenitor during its core collapse. In fact, field as large as $\sim 10^{18}$ G in the core is predicted [8] using scalar virial theorem which is based on Newtonian gravity. At such high matter density, the effect of general relativity is significant and this gives rise to a very strong gravitational force [9] on the star. Consequently, the value of B_m is expected to be further increased above 10^{18} G. Because of highly conducting core, such a high field is frozen in [10] and may not manifest at the surface. The energy of a charged particle changes significantly in the quantum limit if the magnetic field is comparable to or above a critical value $B_m^{(c)}$ [11], and the quantum effects are most pronounced when the particle moves in the lowest Landau level. The interaction of charged particles with strongly quantizing fields has been shown to modify the gross properties of matter on the surface [8,12] as well as in the core of neutron stars [13,14].

Theoretical studies of Fock (exchange) term, relevant to the star surface/crust, in intense magnetic field have been carried out using the simple Thomas-Fermi-Dirac model [15]. In this Letter, we investigate the composition and structure of neutron stars with quark-hadron phase transition under the influence of strong magnetic fields in the Dirac-Hartree-Fock (DHF) approach within a mean-field approximation. This method is rather gen-

eral, so it should be of correspondingly broad interest.

We describe the calculation of the Fock term in presence of a magnetic field in a general formalism within the σ - ω model [14,16]. In a uniform magnetic field B_m along z-axis, the Lagrangian is given by

$$\mathcal{L} = \bar{\psi} [i\gamma_\mu D^\mu - m - g_\sigma \sigma - g_\omega \gamma_\mu \omega^\mu] \psi + \frac{1}{2} (\partial^\mu \sigma)^2 - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{4} (\partial_\mu \omega_\nu - \partial_\nu \omega_\mu)^2 + \frac{1}{2} m_\omega^2 (\omega_\mu)^2, \quad (1)$$

in the usual notation [14]. The general solution for protons is $\psi(\mathbf{r}) \propto \exp(-ie^{HF}t + ip_y y + ip_z z) f_{p_y, p_z}(x)$, where the 4-component spinor at zero temperature $f_{p_y, p_z}(x)$ is of the same form as in Ref. [14], but with the single-particle Hartree energy ϵ^H replaced by the corresponding Hartree-Fock energy ϵ^{HF} . The form of the spinor in a magnetic field (see Ref. [14]) restricts the evaluation of the Fock term to strong fields such that only the zeroth Landau level, $\nu = 0$, is populated. The position dependent part can then be decoupled so that it reduces to the form

$$f_{p_y, p_z}^{\nu=0}(x) = N_{\nu=0} \begin{pmatrix} \epsilon_{\nu=0}^{HF} + p_{vz} \\ 0 \\ -m^* \\ 0 \end{pmatrix} I_{\nu=0; p_y}(x), \quad (2)$$

where $N_{\nu=0} = 1/\sqrt{2\epsilon_{\nu=0}^{HF}(\epsilon_{\nu=0}^{HF} + p_{vz})}$ and $\epsilon_{\nu=0}^{HF} = \epsilon_{p_z}^{HF} - U_0^H - U_0^F(p_z) = \sqrt{p_{vz}^2 + m^{*2}}$. The DHF equation of protons for $\nu = 0$ can then easily be written as

$$[\alpha_z p_z + \beta(m + U^H + U^F)] u(p_z) = \epsilon_{p_z}^{HF} u(p_z), \quad (3)$$

where the effective mass $m^* = m + U_S^H + U_S^F(p_z)$, and $u(p_z)$ is the momentum dependent part of the spinor. For $B_m \neq 0$, the Hartree contribution U^H is given in Ref. [14]. The Fock term U^F in general is given by [17] $\beta U^F(p_z) = \beta U_S^F(p_z) + U_0^F(p_z) + \boldsymbol{\alpha} \cdot \hat{p} U_V^F(p_z)$, where \hat{p} is the unit vector along B_m . For $B_m \neq 0$ the different terms are given by

$$U_S^F(p_z) = \frac{1}{16\pi^2} \int_{-p_F}^{+p_F} dq_z \frac{m^*}{\sqrt{q_{vz}^2 + m^{*2}}} (\mathcal{J}_\sigma - 4\mathcal{J}_\omega),$$

$$U_0^F(p_z) = \frac{1}{16\pi^2} \int_{-p_F}^{+p_F} dq_z (\mathcal{J}_\sigma + 2\mathcal{J}_\omega),$$

$$U_V^F(p_z) = -\frac{1}{16\pi^2} \int_{-p_F}^{+p_F} dq_z \frac{q_{vz}}{\sqrt{q_{vz}^2 + m^{*2}}} (\mathcal{J}_\sigma + 2\mathcal{J}_\omega).$$

Here $q_{vz} = q_z (1 + U_V^F(p_z)/q_z)$ and $\mathcal{J}_a = g_a^2 \exp(l_a^2/2qB_m) \int_0^{\pi/2} d\theta \sec \theta \operatorname{erf}(l_a \sec \theta / \sqrt{2qB_m})$ with $l_a^2 = (p_z - q_z)^2 - (\epsilon_{p_z}^{HF} - \epsilon_{q_z}^{HF})^2 + m_a^2$, and $\operatorname{erf}(x)$ denotes the error function; g_{as} correspond to meson coupling constants with $a \equiv (\sigma, \omega)$. The exchange contribution for $B_m = 0$ is given in Ref. [17,18]. It is straightforward to extend this formalism to calculate the exchange interaction for electrons in the outer crust of a strongly magnetized neutron star [15,19] and also for a magnetized pair Fermi gas [20] at finite temperature and baryon density in exotic stellar objects and cosmology. To explore the quark-hadron phase transition, we demonstrate here the extension of the DHF approach to the calculation of the one-gluon exchange term in the quark phase.

The pure quark phase consisting of u , d and s quarks interacting through one-gluon exchange in local charge neutral and β -equilibrium conditions is described by the bag model [2]. The DHF equations of motion for quarks in strong magnetic field can readily be obtained from Eq. (3) by dropping the σ -meson term and replacing the ω -meson coupling by the quark-gluon coupling, i.e. $g_\omega \rightarrow (g/2)\lambda_{ab}^\gamma$, where $a(b)$ corresponds to the color charge of the outgoing(incoming) quark and λ_s are SU(3) generators. The Hartree term (U^H) vanishes because the color symmetric combination ($\sim \operatorname{tr}\lambda$) does not couple to the gluons. The interaction energy density is then due to exchange term only, and this to order g^2 for each flavor with $B_m \neq 0$ is

$$\mathcal{E}_I^{f;\nu=0} = \frac{q_f B_m}{8\pi^2} \int_{-p_F^f}^{+p_F^f} dp_z \left[U_0^F + \frac{p_{vz}}{\sqrt{p_{vz}^2 + m_f^{*2}}} U_V^F \right],$$

where q_f , m_f^* and p_F^f are the charge, effective mass and Fermi momentum of quark of f th flavor. The QCD coupling constant is defined by $\alpha_c = g^2/4\pi$. The general expression (for all ν) for the total kinetic energy of the quark phase in a magnetic field is

$$\begin{aligned} \mathcal{E}_K^\nu &= \sum_{f=u,d,s} \frac{d_f q_f B_m}{4\pi^2} \sum_{\nu=0}^{\nu_{\max}^{(f)}} g_\nu \Phi(\mu_f^*, m_{f,\nu}^*) \\ &+ \frac{e B_m}{4\pi^2} \sum_{\nu=0}^{\nu_{\max}^{(e)}} g_\nu \Phi(\mu_e, m_{e,\nu}), \end{aligned} \quad (4)$$

where $\Phi(x, y) = x \mathcal{O}_{i,\nu}^{1/2} + y^2 \ln \left\{ \left(x + \mathcal{O}_{i,\nu}^{1/2} \right) / y \right\}$ with $i \equiv f$ or e . The notation in Eq. (4) is same as that

in Ref. [14], but the first term corresponds to those for quarks with $d_f = 3$ and the second term is that for electrons. The total energy density of the pure quark phase is then $\mathcal{E}^q = \mathcal{E}_K^\nu + \sum_f \mathcal{E}_I^{f;\nu=0} + \mathcal{E}_m + B$, where B is the bag constant and the magnetic field energy density is $\mathcal{E}_m = B_m^2 (n_b/n_0)/(8\pi)$. Since the higher order contributions of the density dependent field $B_m(n_b/n_0)$ (given by Eq. (5)) to number densities and chemical potentials of different species are found to be negligible, the magnetic energy density and magnetic pressure have been treated perturbatively. The pressure follows from the relation $P^q = \sum_f \mu_f n_f + \mu_e n_e - \mathcal{E}^q$, where μ_f denotes the quark chemical potential and $n_f = (d_f q_f B_m / 2\pi^2) \sum_{\nu=0}^{\nu_{\max}^{(f)}} g_\nu (\mu_f^{*2} - m_{f,\nu}^{*2})^{1/2}$ is the quark density. The electron density is $n_e = (e B_m / 2\pi^2) \sum_{\nu=0}^{\nu_{\max}^{(e)}} g_\nu (\mu_e^2 - m_{e,\nu}^2)^{1/2}$, and μ_e its chemical potential. The charge neutrality condition, $Q^q = \sum_f q_f n_f - n_e = 0$, and the β -equilibrium conditions, $\mu_d = \mu_u + \mu_e = \mu_s$, can be solved self-consistently together with the effective masses at a fixed baryon number density $n_b^q = (n_u + n_d + n_s)/3$ to obtain the equation of state (EOS) for the deconfined phase. For the ease of numerical computation we, however, add here the one-gluon exchange term perturbatively to energy density and pressure.

To describe pure hadronic matter consisting of neutrons (n), protons (p) and electrons (e), we employ the linear σ - ω - ρ model of Ref. [21] in the relativistic Hartree approach. Neglecting the Fock term in hadron phase is quite justified as the Hartree energy grows like p_F^6 while the exchange energy behaves as p_F^4 [22], so the Fock correction to the EOS is expected to be small [18,22]. The EOS for this phase is obtained by solving self-consistently the effective mass in conjunction with the charge neutrality and β -equilibrium conditions, $Q^h = n_p - n_e = 0$ and $\mu_n = \mu_p + \mu_e$ at a fixed baryon number density n_b^h . Here n_i and μ_i denote the number density and chemical potential; the subscript i refers to n , p and e . The energy density $\mathcal{E}^h = \mathcal{E}_0 + \mathcal{E}_m$ (\mathcal{E}_0 is the contribution to the energy density due to nucleons and electrons [14]) and pressure P^h in this phase are related by $P^h = \sum_i n_i \mu_i - \mathcal{E}^h$.

The mixed phase of hadrons and quarks comprising of two conserved charges, baryon number and electric charge is described following Glendenning [4]. The conditions of global charge neutrality and baryon number conservation are imposed through the relations $\chi Q^h + (1 - \chi)Q^q = 0$ and $n_b = \chi n_b^h + (1 - \chi)n_b^q$, where χ represents the fractional volume occupied by the hadron phase. Furthermore, the mixed phase satisfies the Gibbs' phase rules: $\mu_p = 2\mu_u + \mu_d$ and $P^h = P^q$. The total energy density is $\mathcal{E} = \chi \mathcal{E}^h + (1 - \chi) \mathcal{E}^q$.

In the present calculation the values of the dimensionless coupling constants for σ , ω and ρ mesons determined by reproducing the nuclear matter properties at a saturation density of $n_0 = 0.16 \text{ fm}^{-3}$ are adopted from Ref.

TABLE I. The phase boundaries, u_1 and u_2 , and central densities u_c of stars with maximum masses M_{\max}/M_\odot with and without magnetic fields that undergo a quark-hadron phase transition with interacting quark phase. The corresponding quantities with non-interacting quark phase are shown in parentheses. The variation of magnetic field with density n_b is given by Eq. (5) with $\beta = 0.01$, $\gamma = 3$ for $B_0 = 5 \times 10^{18}$ G and $B_0 = 10^{19}$ G. Calculations are performed for the hadronic Lagrangian of Zimanyi and Moszkowski (ZM) [21] and Serot and Walecka (SW) [16]. The bag constant is $B = 250$ MeV fm $^{-3}$ and the nuclear matter saturation density n_0 is 0.16(0.1484) fm $^{-3}$ for ZM(SW) model.

Hadronic Model	B_m (Gauss)	$u_1 = n_1/n_0$	$u_2 = n_2/n_0$	$u_c = n_c/n_0$	M_{\max}/M_\odot
ZM	0	4.107(3.329)	25.941(17.681)	7.152(7.754)	1.707(1.610)
	$10^{14} - 5 \times 10^{18}$	4.108(3.326)	25.754(17.530)	6.902(7.104)	1.549(1.466)
	$10^{14} - 10^{19}$	4.130(3.312)	25.383(17.085)	5.214(5.016)	1.331(1.268)
SW	0	2.158(1.953)	9.523(7.837)	4.259(4.012)	2.594(2.279)
	$10^{14} - 5 \times 10^{18}$	2.159(1.945)	9.374(7.769)	3.252(3.554)	2.342(2.017)

[4]. The current masses of u and d quarks are taken as $m_u = m_d = 5$ MeV and $m_s = 150$ MeV, and the QCD coupling constant is $\alpha_c = 0.2$. We consider the bag constant $B = 250$ MeV fm $^{-3}$ which corresponds to the lower limit dictated by the requirement that, at low density, hadronic matter is the preferred phase. The variation of the magnetic field B_m with density n_b from the center to the surface of a star is parametrized by the form

$$B_m(n_b/n_0) = B_m^{\text{surf}} + B_0 [1 - \exp \{-\beta(n_b/n_0)^\gamma\}], \quad (5)$$

where the parameters are chosen to be $\beta = 0.01$ and $\gamma = 3$. The maximum field prevailing at the center is taken as $B_0 = 5 \times 10^{18}$ G and the surface field is $B_m^{\text{surf}} \simeq 10^{14}$ G. The number of Landau levels populated for a given species is determined by the field B_m and baryon density [14]. Over the entire density range from the surface to the core, we find that the nucleonic energy density (pressure) dominates substantially over the magnetic field energy density (pressure). Hence matter can sustain such a high field without injecting any instability in the corresponding EOS.

With this parameter set, we show in Table I, the mixed phase boundaries of neutron star for $B_m = 0$ and for $B_m \neq 0$ with interacting quark phase (IQP). To examine the measure of the importance of the exchange contribution, the corresponding results for matter with non-interacting quark phase (NQP) are given in parentheses. The onset of transition is at density $n_1 = u_1 n_0$ and the pure quark phase begins at density $n_2 = u_2 n_0$. For matter with NQP and $B_m = 0$, the boundaries are at $u_1 = 3.329$ and $u_2 = 17.681$. With the inclusion of interaction, the EOS for the quark phase becomes softer and transition to a mixed phase is delayed to $u_1 = 4.107$. At high density, as expected, the EOS for the quark phase is more softer resulting in a larger shift in u_2 to 25.941 compared to that in u_1 . As a consequence, the extent of the mixed phase is increased. With further inclusion of magnetic field $B_0 = 5 \times 10^{18}$ G, the situation is reversed. The EOS for the hadronic sector for $B_m \neq 0$ is found to be softer at low density and it turns out to be stiffer at high density compared to the field-free case [14]. On the

other hand, the EOS for the IQP with $B_m \neq 0$ is found to be considerably stiffer over the entire density range compared to those for $B_m = 0$, with or without the exchange term. Therefore, for $B_0 = 5 \times 10^{18}$ G, the onset of mixed phase occurs at 4.108. The stiff EOS for both hadronic and interacting quark sectors at $B_0 = 5 \times 10^{18}$ G reduce the upper boundary to a density of $u_2 = 25.754$ and thereby also the extent of the mixed phase. Since the EOS of NQP with $B_0 = 5 \times 10^{18}$ G is most stiff of all the cases considered above, the boundaries of the mixed phase are maximally shifted to the lowest baryon densities at $u_1 = 3.326$ and $u_2 = 17.530$, and thus has the smallest mixed phase extent.

For the field-free case, with the appearance of quarks, the neutron and electron abundances are found to decrease since quark matter furnishes both baryon number and negative charge. With $B_0 = 5 \times 10^{18}$ G, apart from reducing the mixed phase extent, the magnetic field enhances the electron fraction in the hadronic sector [14]. Because of charge neutrality condition $n_p = n_e$ in the hadronic phase the proton fraction is also increased. The enhanced electron abundance persists even in the mixed phase. In the mixed phase, with $B_0 = 5 \times 10^{18}$ G, the u -quark abundance is found to be enhanced, while those of d and s quarks remains practically unaltered.

The maximum masses of the stars M_{\max} , obtained by solving the TOV equation [9] are given in Table I for different cases studied. With only nucleons and electrons, without any quark, the maximum masses of the stars are $1.778 M_\odot$ and $1.643 M_\odot$ for $B_m = 0$ and $B_0 = 5 \times 10^{18}$ G. This is a manifestation of the softening of the EOS in magnetic field for these stars [14]. However, the introduction of quarks, which softens the overall EOS, causes the maximum mass to be smaller. For $B_m = 0$, the inclusion of one-gluon exchange in the quark sector causes a delayed appearance of the pure quark phase, and this results in a stiffer overall EOS with a larger maximum mass of $1.707 M_\odot$ in contrast to matter with NQP. Due to further softening of the EOS by $B_0 = 5 \times 10^{18}$ G for matter with IQP, the maximum mass is reduced to a value of $1.549 M_\odot$. The smallest $M_{\max} = 1.466 M_\odot$ is obtained

for the softest EOS with this value of B_0 for matter with NQP. The central densities $u_c = n_c/n_0$ of the maximum mass stars for all cases (see Table I) are less than u_2 and fall within the mixed quark-hadron phase. Therefore the presence of a pure quark phase is precluded.

We have found that when the QCD coupling constant $\alpha_c > 0.25$, “strange matter” becomes negatively charged and requires positrons for overall charge neutrality and therefore would have catastrophic consequences [2].

Calculations are repeated with a higher value of $B_0 = 10^{19}\text{G}$ (see Table I) [23]. Here the maximum masses of the stars for both IQP and NQP are found to be smaller than the observational lower limit of $1.44M_\odot$ imposed by the larger mass of the binary pulsar PSR 1913+16 [24]. Therefore, the quark-hadron phase transition with the present hadronic EOS [21] sets a limit on the maximum value of the field that can be used for a stable system. To estimate the uncertainties in the parametrization of B_m in Eq. (5), calculations are performed with IQP matter for a rapidly increasing value of B_m ($\beta = 0.02$ and $\gamma = 3$) and for a slowly increasing value of B_m ($\beta = 0.005$ and $\gamma = 3$) taking $B_0 = 5 \times 10^{18}\text{G}$. The respective maximum masses are $1.464M_\odot$ and $1.664M_\odot$. Apart from the uncertainties stemming from the parameters of the quark phase (B and α_c), the inadequate knowledge of the hadronic EOS at high densities relevant to a neutron star has important bearing on the phase boundaries in quark-hadron phase transition [6]. To explore this effect, we have also performed calculations with a stiffer hadronic EOS [16]. The phase boundaries for this EOS (see Table I) are found to be shifted at much lower densities; the corresponding maximum masses are much larger than the observed values. Recently similar conclusions have been drawn [6] about the phase transition densities in an effective field theoretic model including non linear scalar and vector meson interactions which soften the hadronic EOS.

Neutron star cooling by URCA process may provide important informations about the interior constitution of the star. It is therefore of interest to see whether cooling by direct URCA process involving quarks can occur for star with quark matter. The decay of d and s quarks are kinematically allowed [25] if they satisfy the respective inequality conditions $p_F^u - p_F^e \leq p_F^d \leq p_F^u + p_F^e$ and $p_F^u - p_F^e \leq p_F^s \leq p_F^u + p_F^e$. With $B_m = 0$, direct URCA process involving u and d quarks is found to occur for $n_b \gtrsim n_0$, while the latter relation is satisfied for $n_b \gtrsim 5.3n_0$. On the other hand, for $B_0 = 5 \times 10^{18}\text{G}$ ($\beta = 0.01$ and $\gamma = 3$) the first inequality condition is satisfied for $n_b \gtrsim n_0$, while the latter occurs for $n_b \gtrsim 2.4n_0$. Thus the threshold density for s quark decay depends sensitively on B_m . Large neutrino luminosity and therefore rapid cooling may serve as an observational means for the presence of strong magnetic field in the inner core of the star, and work in this direction is in progress.

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